Algebraic Topology WS26
Exercise set 5.

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Problem 1. Compute the singular homology of a point.

Problem 2. Show that $H_0(X)$ is the free abelian group on the path components of X.

Problem 3. Does there exist a short exact sequence of abelian groups

$$0 \longrightarrow \mathbb{Z}/4\mathbb{Z} \longrightarrow \mathbb{Z}/8\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z} \longrightarrow \mathbb{Z}/4\mathbb{Z} \longrightarrow 0?$$

If such a sequence exists, give its maps explicitly.

Problem 4. Let

$$0 \longrightarrow U \xrightarrow{i} V \xrightarrow{p} W \longrightarrow 0$$

be a short exact sequence of finite-dimensional vector spaces over a field k. Regard it as a chain complex with U, V, W in degrees $2, 1, 0$. Construct a chain homotopy between the identity map of this complex and the zero map.

Problem 5. Find a way to identify the faces of a tetrahedron in pairs so that the quotient is homeomorphic to S^3. Compute the simplicial homology groups of the resulting Δ-complex with one 3-simplex.

Due date: Monday, 23.11.2026